The implication is that the velocity gradient at the wall is smaller. In practice, the heat transfer coefficients are obtained by analogy with the wall friction coefficients. By using the hydraulic diameter, the heat transfer coefficients are similarly overestimated. By incorporating the new length, the improvement in the heat transfer coefficients can be seen in the Fig. 3, for square and triangular ducts.

In the case of rectangular ducts, the secondary flows exist in strength only in regions close to the shorter sides. Brundrett and Baines [5] have shown that the corner bisector separates the secondary flow cells. Thus, the deviation from the log law is earlier in the corner region. The region of maximum influence along the longer side will be about half a shorter side length, measured from the corner. The rest of the longer side flow can be considered to be analogous to flow between parallel plates. The characteristic length scale for a rectangle should incorporate this by an appropriate weighting for the parallel plate region, as shown in Fig. 4, where $1{\cdot}207D$ is the value of D_L for a square duct.

Finally, for other ducts having symmetry about two axes (e.g. ellipses), it is suggested that they be approximated by straight edges. An average length scale can then be found as for a rectangle.

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ENTRANCE-REGION HEAT TRANSFER FOR LAMINAR FLOW IN POROUS TUBES

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NOMENCLATURE

B_n,	series coefficients for temperature distribution;
d,	tube diameter;
G_n,	series coefficients for local Nusselt number;
Gz_0,	entrance Graetz number, $Re_0 Pr/(x/d)$;
H_n,	eigenfunction for temperature distribution;
k,	thermal conductivity;
Nu,	local Nusselt number; $Nu_{f.d.}$, fully developed Nusselt number;
Pr,	Prandtl number;
q_w,	wall heat flux due to molecular conduction;
r,	radial coordinate; r_w, pipe radius;
Re_0,	axial Reynolds number at $x = 0$, $\bar{u}_0 d/v$;
Re_w,	wall Reynolds number, $v_w d/v$;
T,	temperature; T_0, uniform entrance temperature at $x = 0$; T_w, uniform wall temperature; $T_{\mathfrak{C}}$, centerline temperature;
$\bar{u}_0$,	mean velocity at $x = 0$;
v_w,	wall injection or extraction velocity;
x,	axial location measured from point where step change in wall temperature occurs.

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Greek symbols

- ζ, dimensionless axial coordinate, x/r_w;
- η, dimensionless radial coordinate, r/r_w;
- λ_n, eigenvalue;
- ν, kinematic viscosity.

INTRODUCTION

In a recent study [1], the heat transfer characteristics of laminar flow in constant temperature porous tubes were predicted for thermally fully-developed conditions. These results, which are applicable only beyond some thermal-development length, covered a wide range of fluid Prandtl numbers and wall injection and extraction rates.

In contrast to this prior work, relatively little information has been obtained on the effect of surface mass transfer on thermally developing flows. Yuan and Finkelstein [2] have presented some entrance region heat transfer results. However, these are restricted to asymptotically small values of the wall injection rate. Furthermore, results for even small amounts of wall extraction have not previously been reported.

When there is mass removal at the tube wall, the axial flow is diminished in the downstream direction until the point of complete mass extraction is reached. This point is easily derived from mass conservation. From the results of [1], it is known that thermal development must be obtained before this point is reached. However, because the thermal field is developing during the mass removal, it is possible that fully developed conditions will be reached only just upstream of the point of complete mass extraction. Thus, it is expected that the heat transfer over most of the extraction length will take place in the thermally developing regime.

In the present communication, the effect of surface injection or extraction on the developing temperature field in a fluid following a step change in tube wall temperature is examined. The local Nusselt number and thermal development length are determined for a range of wall injection and extraction rates and several Prandtl numbers; comparisons are made with results obtained for impermeable-walled tubes.

ANALYSIS

The flow is assumed to originate in an adiabatic section of sufficient length to allow the radial and axial velocity profiles to attain their fully developed values. These have been presented and discussed previously by Berman [3]. The mass injection or extraction is normal to the tube wall and is taken to be uniform with axial distance. At $\zeta = 0$, there is a step change in wall temperature (thereafter a constant), and it is desired to calculate the heat transfer in the region $\zeta > 0$.

The mathematical formulation of the problem will not be given in detail here. Portions may be found in [1], and the remainder follows closely the analysis by Terrill [4] for the case of thermally developing flow between two parallel porous plates. It is found that the radial temperature distribution for a uniform entering temperature profile is given by

$$\frac{T(\eta) - T_w}{T_0 - T_w} = \sum_{n=1}^{\infty} B_n H_n(\eta)(1 + 2\,Re_w \zeta / Re_0)^{-\frac{\lambda_n^2}{Re_w Pr}} \quad (1)$$

where $H_n(\eta)$'s are the eigenfunctions for the Sturm–Liouville system (see equation (17) [1]) and the λ_n's are the eigenvalues. Using this result, the Nusselt number based on the wall heat flux due to molecular conduction [i.e. $q_w = -k(\partial T/\partial r)_w$] is given by

$$Nu = \frac{\sum_{n=1}^{\infty} \lambda_n^2 G_n (1 + 4Re_w Pr/Gz_0)^{-\frac{\lambda_n^2}{Re_w Pr}}}{\sum_{n=1}^{\infty} G_n (1 + 4Re_w Pr/Gz_0)^{-\frac{\lambda_n^2}{Re_w Pr}}} - Re_w Pr. \quad (2)$$

It can be seen from equation (2) that the axial dependence of the Nusselt number is contained in the term $(1 + 4Re_w Pr/Gz_0)$, where Gz_0 is the Graetz number based on the entering Reynolds number and is given by $Gz_0 = Re_0 Pr/(x/d)$. For thermally fully developed flow, (2) reduces to $Nu_{f.d.} = \lambda_1^2 - Re_w Pr$.

For impermeable-walled tubes ($Re_w = 0$), the above results reduce to those found for the familar Graetz problem (see Kays [5]). This can be verified by going to the limit as $Re_w \to 0$.

RESULTS

Temperature profiles and local Nusselt number results have been obtained for values of the wall Reynolds number, Re_w, equal to 10, 4, 0, −2, −4·5, and for each of three Prandtl numbers equal to 0·5, 1, 4. The choice of parameters spans the range considered in [1].

An interesting feature of the computed temperature profiles is shown in Fig. 1. In this figure, the effect of wall mass transfer on the axial variation of the dimensionless centerline temperature is shown for $Pr = 1{\cdot}0$. Other Prandtl numbers could have been chosen to illustrate this same behavior. For small values of $1/Gz_0$, the centerline

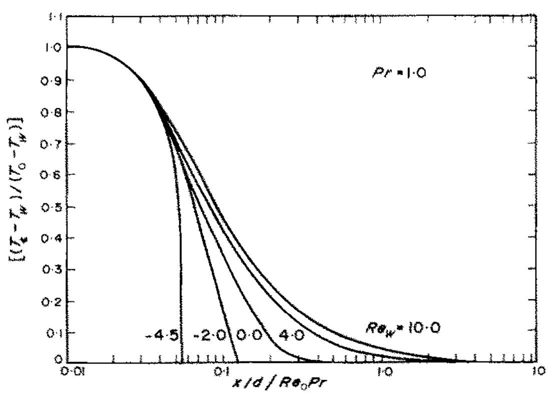

FIG. 1. Variation of centerline temperature in entrance region with injection and extraction.

temperature is equal to the entering temperature, T_0, and remains so until the effect of the step change in T_w is propagated into the fluid by convection and conduction. From this point onward, the centerline temperature approaches T_w with increasing x/d. In the case of the impermeable tube ($Re_w = 0$) and for fluid injection ($Re_w > 0$). T_{C} approaches T_w only asymptotically. However, for the case of fluid extraction ($Re_w < 0$). T_{C} plunges abruptly to T_w, and the curve terminates at a fixed value of $1/Gz_0$. It can be verified that this happens precisely at the point of complete mass extraction. For example, it follows from a mass balance that complete mass extraction occurs when $(1 + 4Re_wPr/Gz_0) = 0$. For $Re_w = -2$ and $Pr = 1$, this corresponds to $1\ Gz_0 = 0{\cdot}125$, the precise value at which $T_{\mathrm{C}} = T_w$ in Fig. 1.

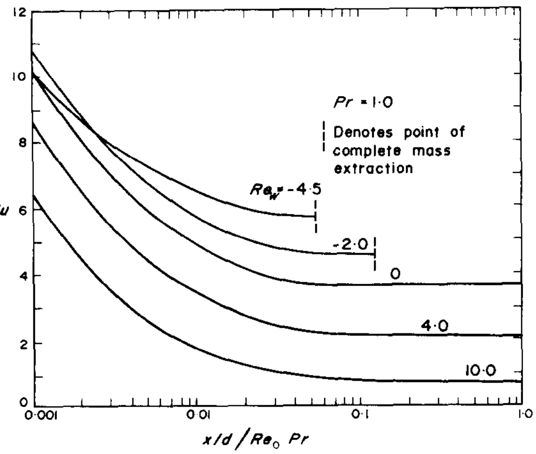

FIG. 2. Variation of entrance region Nusselt number with injection and extraction.

The effect on the developing Nusselt number of the mass transfer Reynolds number and Prandtl number is shown in Figs. 2 and 3. It is noted that fifteen eigenvalues were required to fix Nu to three significant figures at the extreme upstream position corresponding to $1/Gz_0 = 0{\cdot}001$. In each of these figures, the curves for $Re_w < 0$ terminate at the point of complete mass extraction.

It can be seen from Fig. 2 that relative to the results for the impermeable tube ($Re_w = 0$), mass injection ($Re_w > 0$) reduces the Nusselt number while at the same time increases the thermal development length. The opposite trend is generally observed in the case of fluid extraction. However, for the case of $Re_w = -4{\cdot}5$, it is seen in Fig. 2 that the upstream Nusselt number crosses below the curve for $Re_w = -2$. This appears to be due to the singular behavior of the velocity and temperature profiles as Re_w approaches $-4{\cdot}62$. As remarked in [1], the fully developed Nusselt number turns abruptly down towards zero as $Re_w \to -4{\cdot}62$. Indeed, it is believed that exactly at this critical value of Re_w, the entry and fully developed Nusselt numbers are identically zero for all values of $1/Gz_0$ and Pr. Thus, the behavior of the curve for $Re_w = -4{\cdot}5$ appears to be nothing more than a preview of this singular behavior.

It has been verified that all the Nusselt number curves for the case of $Re_w < 0$ do attain their fully developed values prior to the point of complete mass extraction. However, it is apparent from Fig. 3 that this takes place only just upstream of the mass extraction point. For the larger Prandtl number fluids and larger extraction wall Reynolds numbers, there is only a vanishingly small length for which the flow is thermally fully-developed. It is clear that under these conditions, essentially all the heat transfer takes place in the thermal entrance region.

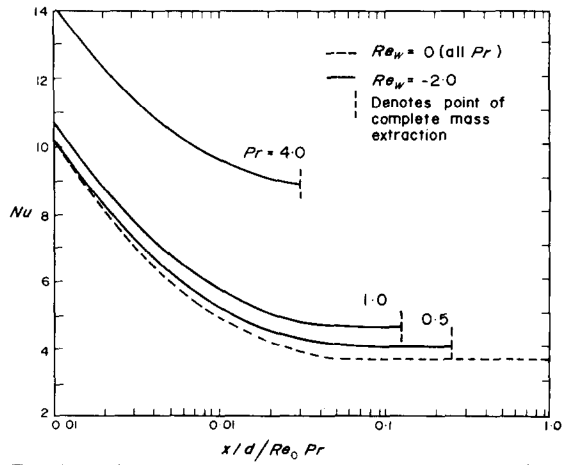

FIG. 3. Variation of entrance region Nusselt number with Prandtl number; $Re_W = -2{\cdot}0$.

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